Roll No. .... Total No. of Questions : 09] [Total No. of Pages : 02

B.Tech. (Sem. - 1st) ENGINEERING MATHEMATICS - I

SUBJECT CODE: BTAM – 101 Paper ID : [A1101] (2011 Batch)

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section – **B & C.**
3) Select atleast **Two** questions from Section – **B & C.**

Section - A

Q1) *(2 Marks each)*

a) Identify the symmetries of the curve $r^2 = \cos\theta$.

b) Find the Cartesian co-ordinates of the point (5, $\tan^{-1}(4/3)$) given in polar co-ordinates.

c) If $u = F(x-y, y-z, z-x)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

d) If $\vec{u}$ is a differentiable vector function of t of constant magnitude, then show that $\vec{u} \cdot \frac{du}{dt} = 0$

e) Change the Cartesian integral $\int_0^1 \int_x^{\sqrt{2x-x^2}} f(x,y)\,dx\,dy$ into an equivalent polar integral.

f) For what values of a,b,c the vector function $\bar{f} = (x + 2y + az)\,\bar{i} - (bx - 3y - z)\,\bar{j} + (4x + cy + 2z)\,\bar{k}$ is irrotational.

g) Give the physical interpretation of divergence of a vector point function.

h) What surface is represented by $\frac{y^2}{2} + \frac{z^2}{3} - \frac{x^2}{2} = 1$?

i) If $x = r\cos\theta$ and $y = r\sin\theta$, then find the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$.

j) Given that F(x,y,z)=0, then prove that $\left(\frac{\partial y}{\partial x}\right)_z \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial x}{\partial z}\right)_y = -1$

Section – B

(8 Marks each)

Q2) a) Show that radius of curvature at any point (x, y) of the hypocycloid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the perpendicular distance from the origin to the tangent at (x, y)

b) Trace the curve $r = 1 + \cos\theta$ by giving all salient features in detail.

Q3) a) Find the area included between the curve $xy^2 = 4a^2\,(2a - x)$ and its asymptote.

b) The curve $y^2\,(a + x) = x^2\,(3a - x)$ is revolved about the axis of x. Find the volume generated by the loop.

Q4) a) If $\theta = t^n e^{\frac{r^2}{4t}}$ then find the value of n that will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$$

b) State Euler's theorem and use it to prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u, \text{ where } u = \tan^{-1}\frac{x^3 + y^3}{x - y}$$

Q5) a) The temperature T at any point (x,y,z) in the space is $T = 400\,x\,y\,z^2$. Use lagrange's multiplier method to find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

b) Expand $x^2y + 3y\text{-}2$ in ascending powers of $x\text{-}1$ and $y + 2$ by using Taylor's theorem.

Section – C *(8 Marks each)*

Q6) a) Evaluate: $\int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$, by changing the order of integration.

b) Find the volume bounded by the cylinder $x^2+y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

Q7) a) Prove that: grad div $\bar{F}$ = curl curl $\bar{F} + \nabla^2 \bar{F}$.

b) Usethe stoke's theorem to evaluate $\int_c [(x + 2y)dx + (x - z)dy + (y - z)dz]$

Where C is the boundary of the triangle with vertices (2,0,0), (0,3,0), and (0,0,6) oriented in the anti-clock wise direction.

Q8) a) Find the directional derivative of $f(x,y,z) = x\,y^2 + yz^3$ at (2,-1,1) in the direction of $\vec{i} + 2\vec{j} + 2\vec{K}$.

b) Find the area lying inside the cardiode $r = 2(1+\cos\theta)$ and outside the circle $r = 2$.

Q9) a) State green's theorem in plane and use it to evaluate $\int_c (y - \sin x)dx + \cos x\,dy$, where C is the triangle enclosed by $y=0$, $x = \frac{\pi}{2}$, and $y = (2/\pi)x$.

b) State Divergence theorem use it to evaluate $\iint_S \vec{F}.\bar{n}d\,s$

where $\vec{F} = (4x^3\vec{i} - x^2y\,\vec{j} + x^2z\,\vec{k}$ and S is the surface of the cylinder $x^2+y^2 = a^2$ bounded by the planes $z = 0$ and $z = b$.