

## THE KINEMATICS OF MACHINERY.

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Most of the models used to illustrate this and the following lecture belong to the Kinematic Collection of the Gewerbe-Akademie in Berlin, and have been designed by Professor Reuleaux, who is the Director of the Academy and a Professor in it. The rest were sent to the Loan Collection by Messrs. Hoff and Voigt of Berlin, and Messrs. Bock and Handrick of Dresden. In essentials there is no difference between the Berlin and the Dresden models. Both have been designed specially for use in instruction in the Kinematics of machinery.

I must first try to explain briefly, but exactly, what I mean by the phrase "Kinematics of machinery." Professor Reuleaux, whose models are before us, defines a machine as "a combination of resistant bodies so arranged that by their means the mechanical forces of nature

can be compelled to do work accompanied by certain determinate motions." The complete course of machine instruction followed in some of the Continental technical schools covers something like the following ground:

First, there is the perfectly general study of machinery, technologically and teleologically. Then there comes what we may call the study of prime movers, which in terms of our definition would be the study of *the arrangements by means of which the natural forces can be best compelled to do the required work*. Then comes the study of what may be called "direct actors," or the direct-acting parts of machinery; in the terms of our definition, *the arrangement of the parts of a machine in such a way as best to obtain the required result*. Next comes what we call machine design; the giving to the bodies forming the machine the requisite quality of resistance. Machine design is based principally on a study of the strength of materials.

One clause of the definition still re-

mains untouched. The machine, we said, does work *accompanied by certain determinate motions*. Corresponding to this we have in machine instruction the *study of those arrangements in the machine by which the mutual motions of its parts, considered as changes of position only, are determined*. The limitation here must be remembered; motion is considered only as a change of position, not taking into account either force or velocity. This is what Professor Willis long ago called the "science of pure mechanism," what Rankine has called the "geometry of machinery," what Reuleaux calls "kinematics," and what I mean now by the "kinematics of machinery."

Ths results of many years' work of Reuleaux in connection with this subject are embodied in his book *Die Theoretische Kinematik*, which I recently had the pleasure of translating, and I shall endeavor to give you an outline of his treatment of the subject. It cannot be more than an outline, as you will readily

understand. The subject is a very large one, and I have had to choose between taking up many branches of it and merely mentioning each, and confining myself to a few points, and going more into detail about them. I have chosen the latter plan, believing that the former would be of little benefit to anybody. It will be easy for those who are sufficiently interested in the matter to follow it up, and to study those parts which I omit, by the aid of the book I have just mentioned. My lecture to-day will be principally theoretical, and to-morrow I shall go more into practical applications. So far as possible, as I have Professor Reuleaux's models before me, I shall endeavor to follow his own order in treating the subject.

I presume you are acquainted, to a certain extent, with the ordinary method of studying "pure mechanism;" the method originated by Monge (1806), developed in Willis' well-known *Principles of Mechanism* (1841), and made popular, to a great extent, by Prof. Goodeve's

capital little text book and others. Each mechanism is studied for and by itself, in general, by the aid of simple algebraic or trigonometric methods, and is spoken of in reference to a certain "conversion" of motion which occurs in it. Thus, we have the conversion of circular into reciprocating motion, the conversion of reciprocating into circular, &c., and simple formulæ express certain relations between the motions of two or more moving points. In this way we know something important about a great number of mechanisms, and arrive at many results which are both useful and interesting. Some things are still left wanting, however; and these things may be summed up in this way:

(1.) We notice at once that we have taken the mechanism as a whole. We do not *analyze* it in any way whatever, and therefore,

(2) We have scarcely any knowledge of its relations with other mechanisms, or (what is quite as important) of the various forms which one and the same

mechanism may take. We shall see presently how extraordinarily various these forms are. We have never a *general* case with special cases derived from it; each case is treated by itself as a special one. Then

(3) The mechanism is studied in general from a point of view which gives us only the conditions of the motion of two points in it, or two portions of it, and is then left. The kinematic conditions of the mechanism *as a whole* remain absolutely untouched.

In such a mechanism as that of an ordinary steam engine, for instance, we study the relative motions of the guide block and the crank, or, I ought, perhaps, to say of the axes of the cross head and of the crank pin. We thus know the motions of two points in the rod which connects those axes, the "connecting rod," but we leave the motions of its other points untouched. It may, of course, be said that these others are of much less practical importance. This is true to some extent, although their practi-

cal importance is greater than might be supposed at first. But in any case these motions must certainly be studied if we are to obtain a *complete* knowledge of the mechanism to which they belong. Any method of study, therefore, which covers all the kinematic conditions of the mechanism, instead of the mechanical conditions of two or three points only, possesses in that respect very great advantages.

The treatment of mechanisms which I shall sketch to you, is intended to remedy some of the defects which I have enumerated. Those of you who have studied modern geometry, side by side with the old methods, will recognize that these defects are somewhat analogous to those of Euclidean geometry. The attempt to remedy them proceeds in lines similar to those of modern geometry, and will eventually, I believe, when more fully worked out, take the same position in its own subject.

Let us, then, look first at the *analysis of mechanisms*. This is none the less

important a matter that its results are so very simple in many cases. A clear understanding of those elementary matters is of great assistance in clearing up difficulties which occur in the more advanced parts of the subject.

In a machine or a mechanism of any kind *the motion of every piece must be absolutely determinate at every instant.* It will be remembered that we are at present considering motion as *change of position* only, not in reference to *velocity*. The motion of change of position *may* be determined by the direction and magnitude of all the external forces which act on the body; the motion is then said to be *free*, but it is obviously impossible to arrange such a condition of things in a machine. The motions may, however, be made absolutely determinate independently of the direction and magnitude of external forces; and in order that this may be the case, the moving bodies, or the moving and fixed bodies as the case may be, must be connected by *suitable geometric forms*. Motion, under



these circumstances, is called *constrained motion*.\*

If I allow a prismatic block to slide down the surface of an inclined plane its motion will be free; it is determined by the combination of external forces which act upon the block. If the block be pressed on one side as it slides, it at once moves sideways, and can only be kept in a straight path if directly the pressure is exerted on the one side an equal and opposite force (or a force which has a resultant with the first in the direction of motion) be caused to act upon it on the other. If, on the other hand, the block be made to slide between accurately-fitting grooves (like a guide block in a machine), inclined at the same angle as the plane, and like it fixed, the block may be pressed sideways or in any other direction, but no alteration in its motion can take place; the motion is "constrained," it can occur

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\* Essentially it does not differ from free motion; the difference really lies in the substitution of *stresses* or *molecular forces*, which are under our complete control, for external forces.

only in the one direction permitted by the guiding grooves. In the one case the external force has to be balanced by another external force; in the other the balancing force is molecular, *i. e.*, is a *stress* and not an external force, and comes at once into play the instant the disturbing force is exerted. The geometric forms which are used in this way to constrain or render determinate the motions in machines are very various, and are chosen in reference to the particular motion required. If every point in a body be required to move in a circle about some fixed axis, a portion of the body is made in the form of a solid of revolution about that axis, and this is caused to "work in" another similar solid; the two forming the familiar pin and eye. If all points of a body be required to move in parallel straight lines we get, similarly for guiding forms, a pair of prisms of arbitrary cross section; a slot and block. If every point of a body be required to move in a helix of the same pitch we use a pair of screws

of that pitch, one solid and one open, for constraining the motion—a screw and nut.

The *general* condition common to these very simple forms is that, in each case, *the path of every point in the moving body is absolutely determined at every instant*, that is to say, the change of position of the moving body is absolutely determinate.

The geometric name for these mutually constraining bodies is *envelopes*, and each one is said to envelope the other. We shall call them (kinematic) *elements*, and the combination of two of them we shall call *a pair of elements*.

Those we have mentioned are special and very familiar and important cases of pairs of elements, which are of great simplicity. They have the common property of surface contact, the one enclosing the other, and are therefore called *closed* or lower pairs of elements. They are, moreover, the only closed pairs which exist. They are, further, the only pairs in which all points of the moving element have *similar* pairs.

Every point of an eye, for instance, moves in a circle about the same axis. If there were attached to it a body of any size or form whatever, all its points would move about the same axis. The "point paths" would all be concentric circles. Again, whatever the external size or shape of a nut, every point in it moves in a helix of the same pitch about the axis of the screw; the point paths, that is, would be similar.

The general condition of determinateness of motion can, however, be fulfilled by an immense number of other pairs of elements. The theory of these is too large a subject to be entered into just now, I must merely direct your attention to the existence of such combinations.

Fig. 1 represents one of the simplest that can be used. Here one of the elements is an equilateral triangle, ABC, the other is the "duangle" RPSQ. The latter moves within the former, touching it always in three points, or rather along three lines. Its motion is just as absolutely determinate as the

motion of a pin in an eye. It is free to move at any instant only about the point in which the three normals to the triangle at the points of contact intersect (as  $Q$  in the Fig.). The models before you show a few of the many forms taken by such pairs of elements. It is worth

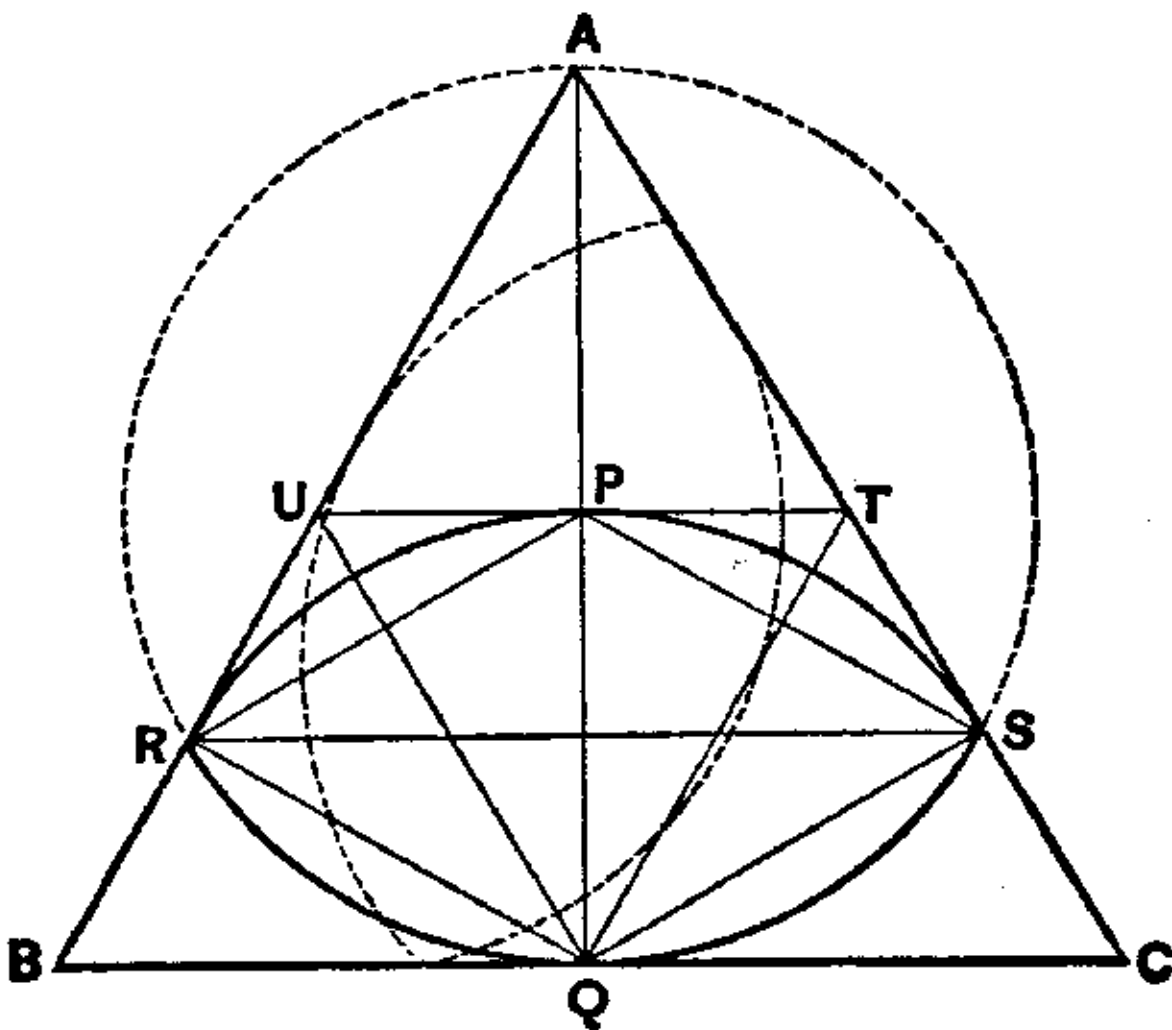


Fig.1

while noticing a few points in which the motions determined by them differ from the motions of the closed pairs. First, as we have already seen, the contact of the elements determining the motion was surface contact in the former

case, while here it takes place only along a finite number of lines. Then the motions of all points in the first case were similar; in these pairs the motions of the points are not similar, but entirely dissimilar, the motion of each point depending entirely upon its position. Fig. 2 shows a few of the point paths of the pair of elements shown in Fig. 1. The strikingly different curves obtained from one pair of elements, according to the choice of the describing point, is too obvious to need further notice.\*

These pairs of elements are called *higher pairs*. They have only a few applications in practice, their interest being chiefly theoretical. From our present point of view their theoretic interest is considerable, because of their exact analogy with the lower pairs.

There is another difference between the two kinds of pairs which deserves notice, for reasons which will be better

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\*The triangle UTQ and the three curves within it, which have  $M_1$  for their center, are point paths. The curve triangle and the duangle shown in thicker lines will be explained further on.

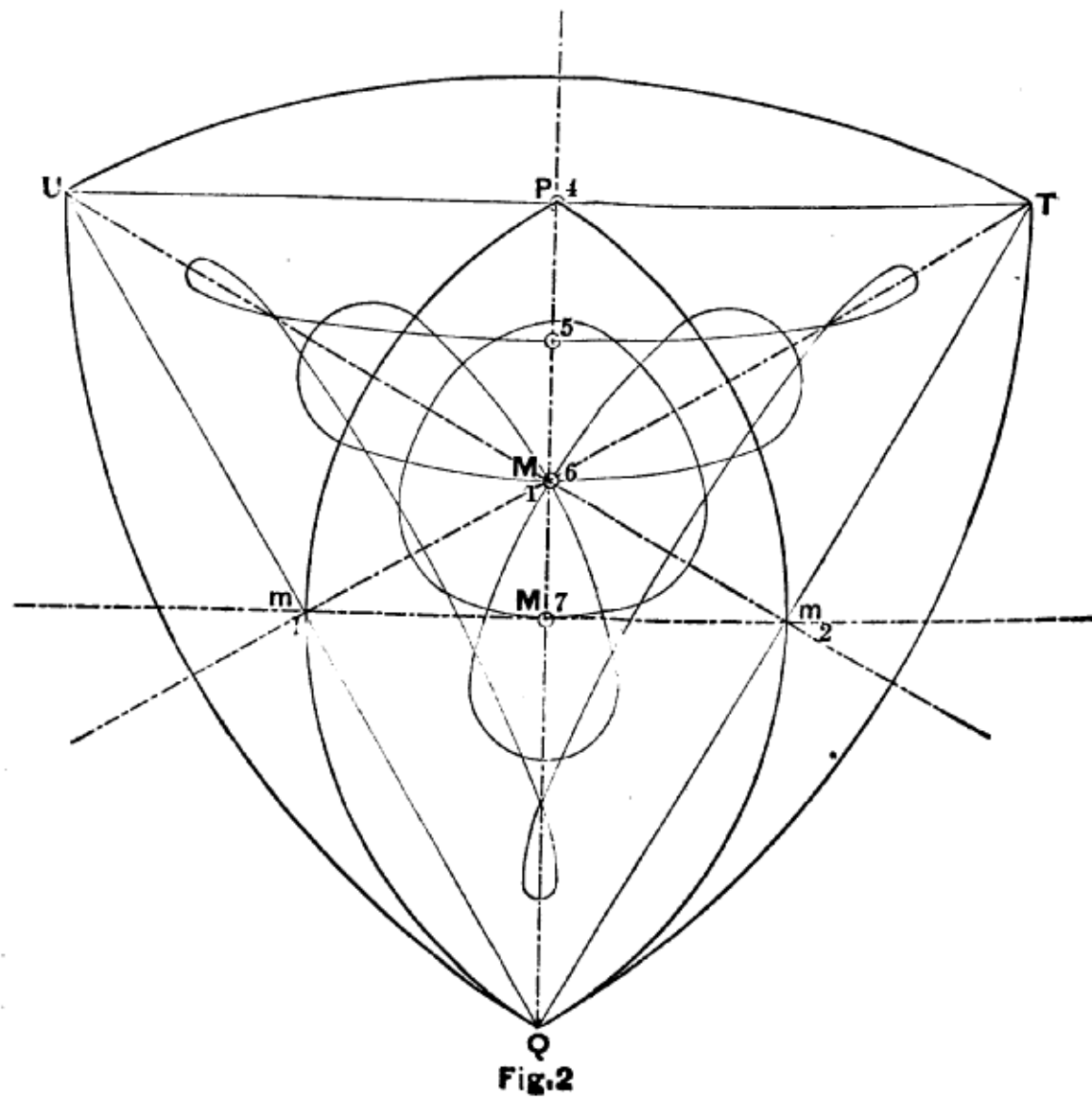


Fig. 2





understood afterwards. The pair of elements determine the relative motion of the two bodies connected by them. If one body be stationary on the floor or the earth, the moving body has the same motion relatively to the floor or earth that it has to the other element. If I move about both bodies in my hand, both have motion relatively to the earth, but the relative motion of the one to the other remains unchanged. It is of course only a case differing in *degree* from the former one, for in the former one both bodies had the motion of the earth itself, while one had the additional motion which I gave it. We may, however, not to be pedantic, speak of anything as "fixed," or "stationary" which has the same motion as the earth.

Now, (in this sense) we may *fix* either element of a pair, and with the lower pairs the *relative motion taking place remains the same* whichever element be fixed. With the higher pairs, on the other hand, the relative motion is altered, and the point paths become en-

tirely different. The point paths of the duangle relatively to the triangle are, for instance, quite different from those of the triangle relatively to the duangle. This change of the fixed element is called the *inversion* of a pair.

The ultimate result of our analysis of mechanisms is then pairs of elements; we cannot go below this. The pairs we have noticed are of two kinds, each having their own definite characteristics. If, now, two or more elements of as many different pairs be joined together we get a combination which is called a (kinematic) *link*. It is obvious that the form of such a link is, kinematically, absolutely indifferent. The choice of its form and material belongs to machine design. It may be brick and mortar, cast iron, timber, as we shall see afterwards, but the fact that this is indifferent, kinematically, cannot be too distinctly kept in mind.

We can make combinations of links by pairing the elements which each contain to partner elements in other links, and

