

Roll No.

Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (Sem.-1st)

ENGINEERING MATHEMATICS-I

Subject Code : AM-101 (2005-2010 Batches)

Paper ID : [A0111]

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTION TO CANDIDATES :

- SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
- SECTION-B & C have FOUR questions each.
- Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
- Select atleast TWO questions from SECTION - B & C.

SECTION-A

I. Write briefly :

(a) If $x^2 = au + bv$, $y^2 = au - bv$, then prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_v = \frac{1}{2}$.

(b) If $u = \log(x^2 + xy + y^2)$, then use Euler's theorem to show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2$$

(c) Express the integral $\int_0^{\infty} e^{-x^4} dx$ in terms of gamma function.

(d) Test the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.



(e) Change the order of integration of $\int_0^{2a} \int_{x^2/4a}^{3a-x} f(x, y) dx dy$.

(f) Test whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$ is absolutely convergent or not.

Explain how?

(g) Use De-Moivre's theorem to express $\sin^5 \theta \cos^2 \theta$ in a series of sines of multiples of θ .

(h) Find the general value i^i .

(i) Verify that $f_{xy} = f_{yx}$, when $f(x, y) = \sin^{-1}(y/x)$.

(j) Obtain the real part of $\tan(A + iB)$.

SECTION-B

2. (a) Trace the curve $y^2(a+x) = x^2(3a-x)$ by giving all salient features in detail.

(b) Find the radius of curvature at the point (r, θ) of the curve $r = a(1 - \cos \theta)$ and show that ρ^2 varies as r^2 .

3. (a) Find the area included between the curves $r = a(1 - \cos \theta)$ and $r = a(1 + \cos \theta)$.

(b) Find the surface of the solid generated by the revolution of the curve $x = a \cos^3 t$, $y = b \sin^3 t$ about y-axis.

4. (a) If $x^x y^y z^z = c$, then show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = - (x \log e x)^{-1}$.

(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin

5. (a) Find the centre of gravity of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant.

(b) Expand $f(x, y) = \sin xy$ in ascending powers of $(x - 1)$ and $(y - \pi/2)$ by using Taylor's theorem up to second degree terms.

SECTION-C

6. (a) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2) dx dy$, by changing into polar co-ordinates.

(b) Find the volume bounded by the co-ordinate planes $x=0, y=0, z=0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

7. (a) Find the centre and radius of the circle

$$x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0, \quad x + 2y + 2z - 20 = 0.$$

(b) Find the equation of a cone whose semi vertical angle is $\pi/4$, has its vertex at origin and axis along the line $x = -2y = z$.

8. (a) If $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$, then prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2} = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$$

(b) If $\sin^{-1}(u + iv) = \alpha + i\beta$ then prove that $\sin^2 \alpha$ and $\cosh^2 \beta$ are the roots of the equation

$$x^2 - (1 + u^2 + v^2)x + u^2 = 0.$$

9. (a) Test for what values of x does the series

$$\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \dots \infty$$

converges/diverges.

(b) Examine the convergence of the following series :

(i)
$$\sum_{n=1}^{\infty} \frac{e^n}{1 + e^{2n}}$$

(ii)
$$\sum_{n=1}^{\infty} \frac{1}{1 + 2^2 + 3^2 + \dots + n^2}$$