

Roll No.

Total No. of Pages : 04

Total No. of Questions : 09

B. Tech. (Sem.-1)

ENGINEERING MATHEMATICS-I

Subject Code : BTAM-101 (2011 Batch)

Paper ID : [A1101]

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. Question-1 is compulsory to attempt, consisting of ten short answer type questions carrying two marks each.
2. Attempt five questions (carrying eight marks each) by selecting at least two questions each from Section A and Section B

1.a) If $w = x^2 + y^2$, $x = r - s$, $y = r + s$, then find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$,

in terms of r , s .

b) Write the Cartesian equivalent of the polar curve $r \cos \left(\theta - \frac{\pi}{2} \right) = 2$.

c) Find the directions in which $f(x,y) = (x^2 + y^2)/2$ increases and decreases most rapidly at the point (1,1).

d) Give the physical interpretation of gradient of a scalar point function.

e) Calculate the outward flux of the field $\vec{F} = x\hat{i} + y^2\hat{j}$ across the square bounded by the lines $x = \pm 1$ and $y = \pm 1$.

f) If \vec{u} is a differentiable vector function of t of constant magnitude, then show that

$$\vec{u} \cdot \frac{d\vec{u}}{dt} = 0.$$

g) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, then find the value of

$$\frac{\partial(u,v)}{\partial(x,y)}$$

h) Find the area enclosed by the lemniscate $r^2 = 4 \cos 2\theta$

i) Change the Cartesian integral $\int_0^2 \int_0^{\sqrt{2r-r^2}} \frac{r}{\sqrt{x^2+y^2}} dy dx$ into an equivalent polar integral

j) Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (xyz) dz dy dx$.

SECTION-A

(8 marks each)

2. a) If ρ_1 and ρ_2 be the radii of curvature at the extremities of a focal chord of the parabola $y^2 = 4ax$, then prove that $(\rho_1)^{-2/3} + (\rho_2)^{-2/3} = (2a)^{-2/3}$.

b) Trace the curve $r = a(1 - \sin \theta)$ by giving all its salient features in detail.

3. a) Find the surface area of the solid formed by revolving the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x -axis.

b) Find the entire length of the cardioid $r = 1 + \cos \theta$ and further show that the upper half is bisected by the ray $\theta = \pi/3$.

4. a) State Euler's theorem and use it to prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0, \text{ whenever } u = x\phi\left(\frac{y}{x}\right) + \varphi\left(\frac{y}{x}\right)$$

b) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}.$$

5. a) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction.

b) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ by Taylor's theorem.

SECTION-B (8 marks each)

6. a) Evaluate : $\int_0^1 \int_x^1 \sin y^2 dy dx$ by changing the order of integration.

b) Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = a$.

7. a) Prove the identity :

$$\nabla \times (\vec{F} \times \vec{G}) = \vec{F}(\nabla \cdot \vec{G}) - \vec{G}(\nabla \cdot \vec{F}) + (\vec{G} \cdot \nabla)\vec{F} - (\vec{F} \cdot \nabla)\vec{G}$$

b) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where C is circular path

$$x^2 + y^2 = a^2, z = 0.$$

8. a) Verify Stoke's for a vector field defined by

$\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ taken around the rectangle bounded by the lines $x = 0, x = a, y = 0, y = b$.

b) Test whether the vector

$\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$, is irrotational or not?

9. a) State Green's theorem in plane and use it to evaluate

$$\int_C (2x^2 - y^2) dx + (x^2 + y^2) dy$$

where C is the boundary of the region enclosed by the x -axis and the upper half of the circle $x^2 + y^2 = a^2$.

b) Find the rate of change of $f(x, y, z) = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point $(1, 1, 1)$.