

May 2010

Roll No.

Total No. of Questions : 09]

[Total No. of Pages : 04

B.Tech. (Sem. • 1st)

ENGINEERING MATHEMATICS - I •

SUBJECT CODE : AM - 101 (2K4 & Onwards)

Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

- 1) Section - A is **Compulsory**.
- 2) Attempt any **Five** questions from Section - B & C.
- 3) Select at least **Two** questions from Section - B & C.

Section - A

(Marks : 2 Each)

Q1)

- a) Find the entire length of the cardioid

$$r = a(1 + \cos\theta)$$

- b) Use Euler's theorem to show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u,$$

where $u = e^{x^2 + y^2}$.

- c) If $x = r \cos\theta$, $y = r \sin\theta$, then show that

$$\frac{\partial(r, \theta)}{\partial(x, y)} = \frac{1}{r}.$$

- d) Find the percentage error in the area of an ellipse when an error of +1 percent is made in measuring the major and minor axes.

- e) For what value(s) of k will the plane

$$x - 2y - 2z = k \text{ touch the sphere}$$

$$x^2 + y^2 + z^2 - 2x + 4y - 6z + 5 = 0.$$

- f) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$, by changing to polar co-ordinates.

- g) State Cauchy root test and use it to test the convergence of the series:

$$\sum \left(\frac{n}{n+1} \right)^{n^2}$$

- h) Examine the convergence of

$$\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \dots$$

- i) If $\sin(A + iB) = x + iy$, then prove that

$$\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$$

- j) Find all the values of $(1 - i)^{1+i}$.

Section - B

(Marks : 8 Each)

- Q2) (a) Trace the curve $y^2(2a - x) = x^3$, by giving all its features in detail.

- (b) Prove that the radius of curvature of the curve $r^n = a^n \cos n\theta$, $n = 1, 2, \dots$

at any point (r, θ) is $\frac{a^n}{(n+1)r^{n-1}}$.

- Q3) (a) Find the area of one loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$.

- (b) Obtain the volume of the spindle-shaped solid generated by revolving the asteroïd $x^{2/3} + y^{2/3} = a^{2/3}$ about the x -axis.

- Q4) (a) If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right)^2 u = -\frac{9}{(x+y+z)^2}$$

- (b) Transform the Laplacian equation $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$, into polar form.

- Q5) (a)** Expand $f(x, y) = x^2y + 3y - 2$ in ascending powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem for several variables.
- (b)** Prove that a rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

Section - C

(Marks : 8 Each)

- Q6) (a)** A plane passes through a fixed point (a, b, c) and cuts axes in A, B and C. Show that locus of the center of the sphere OABC is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2.$$

- (b)** Find the equation of the cone whose vertex is $(1, 2, 3)$ and guiding curve is the circle.

$$x^2 + y^2 + z^2 = 4, \quad x + y + z = 1.$$

- Q7) (a)** Evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}} dy dx.$$

by changing the order of integration.

- (b)** Establish the result.

$$\int_a^b (x-a)^{m-1} (b-x)^{n-1} dx = (b-a)^{m+n-1} \beta(m, n),$$

m, n, a, b are positive constants.

- Q8) (a)** Discuss the Convergence/Divergence of the series

(i) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$

(ii) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

- (b)** Find the radius and interval of convergence of the series.

$$\sum \frac{(3x + 1)^{n+1}}{2n + 2}$$

Further, for what values of x (if any) does the series converges

(i) absolutely

(ii) conditionally.

Q9) (a) Find the sum of the series:

$$\sin^2 \theta - \frac{1}{2} \sin 2\theta \sin^2 \theta + \frac{1}{3} \sin 3\theta \sin^3 \theta - \dots - \infty.$$

(b) Use De-Moivre's theorem to solve the equation $(x - 1)^5 + x^5 = 0$.

